

Figure 12.16 Schematic of the circulatory system. Pressure difference is created by the two pumps in the heart and is reduced by resistance in the vessels. Branching of vessels into capillaries allows blood to reach individual cells and exchange substances, such as oxygen and waste products, with them. The system has an impressive ability to regulate flow to individual organs, accomplished largely by varying vessel diameters.

Each branching of larger vessels into smaller vessels increases the total cross-sectional area of the tubes through which the blood flows. For example, an artery with a cross section of  $1 \text{ cm}^2$  may branch into 20 smaller arteries, each with cross sections of  $0.5 \text{ cm}^2$ , with a total of  $10 \text{ cm}^2$ . In that manner, the resistance of the branchings is reduced so that pressure is not entirely lost. Moreover, because  $Q = A\overline{v}$  and A increases through branching, the average velocity of the blood in the smaller vessels is reduced. The blood velocity in the aorta (diameter = 1 cm) is about 25 cm/s, while in the capillaries ( $20\mu$ m in diameter) the velocity is about 1 mm/s. This reduced velocity allows the blood to exchange substances with the cells in the capillaries and alveoli in particular.

# 12.5 The Onset of Turbulence

Sometimes we can predict if flow will be laminar or turbulent. We know that flow in a very smooth tube or around a smooth, streamlined object will be laminar at low velocity. We also know that at high velocity, even flow in a smooth tube or around a smooth object will experience turbulence. In between, it is more difficult to predict. In fact, at intermediate velocities, flow may oscillate back and forth indefinitely between laminar and turbulent.

An occlusion, or narrowing, of an artery, such as shown in Figure 12.17, is likely to cause turbulence because of the irregularity of the blockage, as well as the complexity of blood as a fluid. Turbulence in the circulatory system is noisy and can sometimes be detected with a stethoscope, such as when measuring diastolic pressure in the upper arm's partially collapsed brachial artery. These turbulent sounds, at the onset of blood flow when the cuff pressure becomes sufficiently small, are called *Korotkoff sounds*. Aneurysms, or ballooning of arteries, create significant turbulence and can sometimes be detected with a stethoscope. Heart murmurs, consistent with their name, are sounds produced by turbulent flow around damaged and insufficiently closed heart valves. Ultrasound can also be used to detect turbulence as a medical indicator in a process analogous to Doppler-shift radar used to detect storms.



Figure 12.17 Flow is laminar in the large part of this blood vessel and turbulent in the part narrowed by plaque, where velocity is high. In the transition region, the flow can oscillate chaotically between laminar and turbulent flow.

An indicator called the **Reynolds number**  $N_{\rm R}$  can reveal whether flow is laminar or turbulent. For flow in a tube of uniform diameter, the Reynolds number is defined as

$$N_{\rm R} = \frac{2\rho v r}{\eta}$$
 (flow in tube), [12.53]

where  $\rho$  is the fluid density, v its speed,  $\eta$  its viscosity, and r the tube radius. The Reynolds number is a unitless quantity. Experiments have revealed that  $N_{\rm R}$  is related to the onset of turbulence. For  $N_{\rm R}$  below about 2000, flow is laminar. For  $N_{\rm R}$  above about 3000, flow is turbulent. For values of  $N_{\rm R}$  between about 2000 and 3000, flow is unstable—that is, it can be laminar, but small obstructions and surface roughness can make it turbulent, and it may oscillate randomly between being laminar and turbulent. The blood flow through most of the body is a quiet, laminar flow. The exception is in the aorta, where the speed of the blood flow rises above a critical value of 35 m/s and becomes turbulent.

# EXAMPLE 12.9

## **Is This Flow Laminar or Turbulent?**

Calculate the Reynolds number for flow in the needle considered in <u>Example 12.8</u> to verify the assumption that the flow is laminar. Assume that the density of the saline solution is  $1025 \text{ kg/m}^3$ .

### Strategy

We have all of the information needed, except the fluid speed *v*, which can be calculated from  $\overline{v} = Q/A = 1.70$  m/s (verification of this is in this chapter's Problems and Exercises).

### Solution

Entering the known values into  $N_{\rm R} = \frac{2\rho v r}{n}$  gives

$$\begin{split} N_{\rm R} &= \frac{2\rho v r}{\eta} \\ &= \frac{2(1025 \text{ kg/m}^3)(1.70 \text{ m/s})(0.150 \times 10^{-3} \text{ m})}{1.00 \times 10^{-3} \text{ N} \cdot \text{s/m}^2} \\ &= 523. \end{split}$$



### Discussion

Since  $N_{\rm R}$  is well below 2000, the flow should indeed be laminar.

## **Take-Home Experiment: Inhalation**

Under the conditions of normal activity, an adult inhales about 1 L of air during each inhalation. With the aid of a watch, determine the time for one of your own inhalations by timing several breaths and dividing the total length by the number of breaths. Calculate the average flow rate Q of air traveling through the trachea during each inhalation.

The topic of chaos has become quite popular over the last few decades. A system is defined to be *chaotic* when its behavior is so sensitive to some factor that it is extremely difficult to predict. The field of *chaos* is the study of chaotic behavior. A good example of chaotic behavior is the flow of a fluid with a Reynolds number between 2000 and 3000. Whether or not the flow is turbulent is difficult, but not impossible, to predict—the difficulty lies in the extremely sensitive dependence on factors like roughness and obstructions on the nature of the flow. A tiny variation in one factor has an exaggerated (or nonlinear) effect on the flow. Phenomena as disparate as turbulence, the orbit of Pluto, and the onset of irregular heartbeats are chaotic and can be analyzed with similar techniques.

# 12.6 Motion of an Object in a Viscous Fluid

A moving object in a viscous fluid is equivalent to a stationary object in a flowing fluid stream. (For example, when you ride a bicycle at 10 m/s in still air, you feel the air in your face exactly as if you were stationary in a 10-m/s wind.) Flow of the stationary fluid around a moving object may be laminar, turbulent, or a combination of the two. Just as with flow in tubes, it is possible to predict when a moving object creates turbulence. We use another form of the Reynolds number  $N'_R$ , defined for an object moving in a fluid to be

$$N'_{\rm R} = \frac{\rho v L}{\eta}$$
 (object in fluid), [12.55]

where L is a characteristic length of the object (a sphere's diameter, for example),  $\rho$  the fluid density,  $\eta$  its viscosity, and v the object's speed in the fluid. If  $N'_{\rm R}$  is less than about 1, flow around the object can be laminar, particularly if the object has a smooth shape. The transition to turbulent flow occurs for  $N'_{\rm R}$  between 1 and about 10, depending on surface roughness and so on. Depending on the surface, there can be a *turbulent wake* behind the object with some laminar flow over its surface. For an  $N'_{\rm R}$  between 10 and  $10^6$ , the flow may be either laminar or turbulent and may oscillate between the two. For  $N'_{\rm R}$  greater than about  $10^6$ , the flow is entirely turbulent, even at the surface of the object. (See Figure 12.18.) Laminar flow occurs mostly when the objects in the fluid are small, such as raindrops, pollen, and blood cells in plasma.

# EXAMPLE 12.10

### **Does a Ball Have a Turbulent Wake?**

Calculate the Reynolds number  $N'_{\rm R}$  for a ball with a 7.40-cm diameter thrown at 40.0 m/s.

### Strategy

We can use  $N'_{\rm R} = \frac{\rho v L}{\eta}$  to calculate  $N'_{\rm R}$ , since all values in it are either given or can be found in tables of density and viscosity.

### Solution

Substituting values into the equation for  $N'_{\rm R}$  yields

$$N'_{R} = \frac{\rho vL}{\eta} = \frac{(1.29 \text{ kg/m}^{3})(40.0 \text{ m/s})(0.0740 \text{ m})}{1.81 \times 10^{-5} 1.00 \text{ Pa} \cdot \text{s}}$$
  
= 2.11 × 10<sup>5</sup>.

### Discussion

This value is sufficiently high to imply a turbulent wake. Most large objects, such as airplanes and sailboats, create significant turbulence as they move. As noted before, the Bernoulli principle gives only qualitatively-correct results in such situations.

One of the consequences of viscosity is a resistance force called **viscous drag**  $F_V$  that is exerted on a moving object. This force typically depends on the object's speed (in contrast with simple friction). Experiments have shown that for laminar flow ( $N'_R$  less than about one) viscous drag is proportional to speed, whereas for  $N'_R$  between about 10 and  $10^6$ , viscous drag is proportional to speed squared. (This relationship is a strong dependence and is pertinent to bicycle racing, where even a small headwind causes significantly increased drag on the racer. Cyclists take turns being the leader in the pack for this reason.) For  $N'_R$  greater than  $10^6$ , drag increases dramatically and behaves with greater complexity. For laminar flow around a sphere,  $F_V$  is proportional to fluid viscosity  $\eta$ , the object's characteristic size L, and its speed v. All of which makes sense—the more viscous the fluid and the larger the object, the more drag we expect. Recall Stoke's law  $F_S = 6\pi r \eta v$ . For the special case of a small